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University Examinations 2022/2023

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICAL SCIENCES

SPH 7103: MATHEMATICAL PHYSICS

DATE: FEBRUARY 2023 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Given the Riemann tensor $(R(X,Y,Z)$, state the Jacobi identify in terms of vector fields X,Y,Z (2 marks)

b) Show that the Bianchi identity $\nabla_{\sigma} R^{\mu}_{\nu\beta\sigma} + \nabla_{\beta} R^{\mu}_{V\sigma\alpha} + \nabla_{\beta} R^{\mu}_{V\sigma\alpha} = 0$ (8 marks)

c) Consider a unitary group representation of

$$|\uparrow> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\downarrow> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

As spin $\frac{1}{2}$ up and $\frac{1}{2}$ down respectively. Show that special unitary group

$$\begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}$$

SU(2) with unitary determinant can give rise to Pauli matrices (10 marks)

d) Transform the harmonic oscillator differential equation

$y'' + w^2 y(x) = 0$ subject to $y(0) = 0; y(b) = 0$ into a Fredholm integral equation of second kind $y(x) = \omega^2 \int_0^b K(x,t) y(t) dt$ (10 marks)

QUESTION TWO (20 MARKS)

a) Solve the D'alermbert's 1-dimensional wave equation $u_{xx} + \frac{1}{c^2} u_{tt} = 0$ subject to $u(x,0) = f(x),; u_t(x,0) = g(x)$ by canonical transform indicating clearly the directions of the waves (10 marks)

b) The temperature θ in the semi-infinite rod $0 < x > \infty$ is determined by the equation (10 marks)

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}$$

Subject to $\theta = \begin{cases} 0 & t = 0, x > 0 \\ \theta_0 & t > 0, x = 0 \end{cases}$

By making use of sine Fourier transform

$$\int_0^\infty \frac{\varsigma \sin \varsigma x d\varsigma}{\varsigma^2 + a^2} d\varsigma = \frac{\pi}{2} e^{-ax}$$

Show that $\theta(x,t) = \frac{2}{\pi} \theta_0 \int_0^\infty \frac{\sin \varsigma x \left(1 - e^{k\varsigma^2 t}\right)}{\varsigma} d\varsigma$

QUESTION THREE (20 MARKS)

Use Eistein's equivalence principle to derive the equation of motion in curved spacetime (Geodesic equation) (20 marks)

QUESTION FOUR (20 MARKS)

a) State the residue Theorem (2 marks)

b) Evaluate the integral $l = \int_0^{\infty} \frac{\sin x}{x^2 - \sigma^2} dx$ (8 marks)

c) Hence show that the quantum mechanical scattering equation

$l(\sigma) = \int_0^{\infty} \frac{\sin x}{x^2 - \sigma^2} dx, \quad \sigma \in R$ leads to the solution of the form $l_+ = \pi e^{i\sigma}$ and $l_- = \pi e^{-i\sigma}$ of the outgoing and incoming wave respectively given that σ takes the form of $e^{i\sigma}$

(10 marks)