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University Examinations 2022/2023

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICAL SCIENCES

SPH 7102: CLASSICAL MECHANICS

DATE: FEBRUARY 2023 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Differentiate between sceleronomous and rhenomous constraints giving an example of each (4 marks)

b) State two problems posed by constraints and how to overcome them (4 marks)

c) Of what significance is variational principle formulation? (2 marks)

d) If L is a lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that

$L = L^{'} + \frac{dF}{dt}\left(q_{1,} q_2, ...q_n, t\right)$ also satisfies Lagrange's equation, where F I arbitrary, but differentiable functions of its arguments (10 marks)

e) Show that for a multiparticle system, angular momentum is broken into two parts corresponding to the centre of mass and about the centre of mass (10 marks)

QUESTION TWO (20 MARKS)

a) A particle of mass m moves in one-dimension such that it has a Lagrangian $L = \frac{m^2 x^4}{12} + mx^2 V(x) - V^2(x)$ where V is some differentiable function of x. Find the equation of motion for $x(t)$ and describe the physical nature of the system based on this equation (10 marks)

b) A particle of mass m is constrained to move under gravity without friction on the inside of a paraboloid of revolution whose axis is vertical. Find the one-dimensional problem equal to its motion. What is the condition on the particle's initial velocity to produce circular motion? Find the period of small oscillations about this circular motion (10 marks)

QUESTION THREE (20 MARKS)

Obtain the solution to the perturbed linear harmonic oscillator

$H = \frac{p^2}{2m} + \frac{1}{2} mw^2 x^2$ where $\frac{p^2}{2m}$ is the unperturbed Hamilton and $\frac{1}{2} mw^2 x^2$ is the perturbation

(20 marks)

QUESTION FOUR (20 MARKS)

a) Define what is mcant by virtual displacement of a system (2 marks)

b) Derive the principle of virtual work (4 marks)

c) Show that the principle of virtual work leads to DAlemberts principle (4 marks)

d) Show that DAlmberts principle leads to Euler-Langrange equation in terms of kinetic energy and component of generalized force (6 marks)

e) Apply the derived Euler-Lagrange equations above on the motion of a particle in plane Cartesian coordinates to obtain the equation of motion (4 marks)