# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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**UNIVERSITY EXAMINATIONS 2022/2023**

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PHYSICS

**SPH 7101: QUANTUM MECHANICS**

**DATE: FEBRUARY 2023** **TIME: 3 HOURS**

**INSTRUCTIONS: Answer Question ONE and any other TWO questions.**

**QUESTION ONE (30 MARKS)**

a) Use Heisnbeg's equatum of motion to obtain the time evolution for position $\hat{x}$ and linear momentum $\hat{p}$ operator of a one-dimensional harmonic oscillator whose Hamiltonian is $\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$ (4 Marks)

b) Consider a wavefunction $\Psi(x,t) = \left(a_1 e^{\frac{ipx}{\hbar}} + a_2 e^{\frac{-ipx}{\hbar}}\right) e^{\frac{-ip^2 t}{2m\hbar}}$

Find the probability current corresponding to this wavefunction. (5 Marks)

c) Consider the wave function $\psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$ where A, $\lambda$ and $\omega$ are positive real constants

i. Normalize $\psi$ (5 Marks)

ii. Determine the expected values of $x$ and $x^2$ (4 Marks)

d) Consider a particle of mass $m$ confined in an infinite one-dimensional potential well of width L such that $v(x) = \begin{cases} 0 & -\frac{L}{2} \le x \le \frac{L}{2} \\ \infty & otherwise \end{cases}$

Find:

i. The eigenstates of the Hamiltonian i.e. stationary states. (3 Marks)

Schrödinger equation

i. The corresponding eigenfunctions. (3 Marks)

e) A particle is described by the wavefunction $\psi(x)=\left(\frac{\pi}{a}\right)^{-\frac{1}{4}} e\frac{-ax^2}{2}$

f) Calculate $\Delta x$ and $\Delta p$ and verify the uncertainty relation. (6 Marks)

**QUESTION TWO (20 MARKS)**

Obtain the solution of the Schrödinger Equation for a free particle confined to move along positive x,y,z axes in a volume abc where a,b and c are the sides of the box. Further show that its energy values are continuous. (20 Marks)

**QUESTION THREE (20 MARKS)**

An incident particle of charge $q_1$ and kinetic energy E scatters off a heavy stationary particle of charge $q_2$

a) Derive the formula relating the impact parameter and the scattering angle. (10 Marks)

b) Determine the usual scattering cross-section. (5 Marks)

c) Show that the total cross-section for Rutherford scattering is infinite. We say that the $\frac{1}{r}$ potential has 'infinite range' (5 Marks)

**QUESTION FOUR (20 MARKS)**

Two identical spin-zero bosons are placed in an infinite square well. They interact weakly with one another, via the potential.

$V(x_1, x_2) = -aV_0\delta(x_1 - x_2)$

Where $V_0$ is a constant with the dimensions of energy, and $a$ is the width of the well.

a) Derive first order time-independent perturbation theory. (8 Marks)
b) Ignoring the interaction between the particles, find the ground state and the first excited state – both the wave functions and the associated energies. (5 Marks)
c) Use first-order perturbation theory to estimate the effect of the particle – particle interaction on the energies of the ground state and the first excited state. (7 Marks)