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UNIVERSITY EXAMINATIONS 2022/2023

THIRD YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION
FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND PHYSICS

SPH 3352: MATHEMATICAL PHYSICS III

DATE: AUGUST 2023

TIME: 2 HOURS

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) Given the Riemann tensor $(R(X, Y, Z))$, state the Jacobi identity in terms of vector fields X, Y, Z . (2 Marks)
- b) Show that the Bianchi identity $\nabla_{\sigma} R^{\mu}_{\nu\alpha\beta} + \nabla_{\alpha} R^{\mu}_{\nu\beta\sigma} + \nabla_{\beta} R^{\mu}_{\nu\sigma\alpha} = 0$ lead to Einstein field equations. (8 Marks)
- c) i. Differentiate between Isomorphism and Homomorphism (2 Marks)
- ii. Show that the complex number $u^3 = e^{\frac{2\pi i}{3}}$ forms a cyclic group hence write down the group elements. (8 Marks)
- d) transform the harmonic oscillator differential equation $y'' + \omega^2 y(x) = 0$ subject to $y(0) = 0; y(b) = 0$ into a Fredholm integral equation of second kind $y(x) = \omega^2 \int_0^b K(x, t) y(t) dt$. (10 Marks)

QUESTION TWO (20 MARKS)

Use the method of separation of variables to determine the steady-state temperature in a uniform solid sphere of radius unity when one half of the surface is kept at a constant temperature 0°C and the other half at the constant temperature 1°C .

QUESTION THREE (20 MARKS)

- a) Define a group theory (8 Marks)
- b) Show that the nonzero set of real numbers form a group (2 Marks)

QUESTION FOUR (20 MARKS)

- a) State the residue theorem. (3 Marks)
- b) Using residues theorem, find the residues of $I = \int_0^{\infty} \frac{x^2 dx}{(x^2+a)(x^2+4)^2}$ (10 Marks)
- c) Hence evaluate the integral (7 Marks)