



# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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## UNIVERSITY EXAMINATIONS 2022/2023

THIRD YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION  
FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND PHYSICS AND  
BACHELOR OF EDUCATION SCIENCE

### SPH 3350: QUANTUM MECHANICS I

DATE: AUGUST 2023

TIME: 2 HOURS

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

#### QUESTION ONE (30 MARKS)

- a) Define the following terms as used in quantum mechanics
- Stationary states (2 Marks)
  - Coherent states (2 Marks)
  - Scattering states (2 Marks)
  - Bound states (2 Marks)
  - Tunneling (2 Marks)
- b) Derive the fundamental quantization  $[\hat{x}, p_x] = i\hbar$  (5 Marks)
- c) Obtain the commutation relation  $[\hat{a}, \hat{a}^+]$  for a linear harmonic oscillator when  $\hat{a}$  and  $\hat{a}^+$  are the annihilation and creation operators respectively. (5 Marks)
- d) Prove that the Hamiltonian operator is symmetric (10 Marks)

#### QUESTION TWO (20 MARKS)

Consider a one dimensional harmonic oscillator with orthogonal basis  $e_0, e_1, e_2, e_3, \dots$ . If the position, momentum and Hamiltonian of the system is given by

$$\hat{p} = \sqrt{\frac{m\omega\hbar}{2}}(\hat{a} + \hat{a}^+)$$

$$\hat{x} = i\sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^+)$$

$$\hat{H}_{en} = \left(n + \frac{1}{2}\right) e_n,$$

Verify that Heisenberg's uncertainty relation is satisfied

### QUESTION THREE (20 MARKS)

A particle of mass  $m$  is in a state  $\varphi(x, t) = Ae^{-a\left(\frac{mx^2}{\hbar} + it\right)}$  where  $A$  and  $a$  are constants .

- Calculate  $A$  (3 Marks)
- For what potential  $v(x)$  does  $\varphi$  satisfy the schrodinger equation. (4 Marks)
- Calculate the uncertainty in  $\hat{x}$  and  $\hat{p}$ . (7 Marks)
- Is Heisenberg uncertainty relation satisfied? (6 Marks)

Note: You may use the elation  $\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} = \frac{\pi}{\alpha} e^{-\frac{\beta^2}{4\alpha}}$

### QUESTION FOUR (20 MARKS)

Consider the Gaussian distribution  $\rho(x) = Ae^{-\lambda(x-a)^2}$  where  $A, a, \lambda$  are positive constants.

Find

- The value of  $A$  (5 Marks)
- $\langle x \rangle$  (5 Marks)
- Uncertainty in position measurement (5 Marks)
- Sketch the graph of  $\rho(x)$  (5 Marks)