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UNIVERSITY EXAMINATIONS 2022/2023

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND PHYSICS AND BACHELOR OF EDUCATION SCIENCE

SPH 3254: MATHEMATICS PHYSICS II

DATE: AUGUST 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) i. Show that the equation $(x^2 + y^2)dx + 2xydy = 0$ is homogeneous. (2 Marks)

ii. Solve the above homogenous equation. (4 Marks)

b) Use the method of integrating factors to solve $coshxdy + (ysinhx = e^x)dx = 0$.

(5 Marks)

c) Use the method of variation of parameter method to solve $y''(x) + 2y'(x) - 3y = 8$

(6 Marks)

d) Write Laurent series that represent the function $f(z) = \frac{-1}{(z-1)(z-2)}$ throughout the annulus $1 < |z| < 2$. (6 Marks)

e) Find the fourier cosine transform of $f(x) = e^{-ax^2}, a > 0$. (7 Marks)

QUESTION TWO (20 MARKS)

a) Give the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ subject to $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$

Transform it into its canonical form. (10 Marks)

b) The temperature of a hot rod governed by the heat equation $\frac{\partial^2 U}{\partial x^2} = \frac{1}{k}\frac{\partial U}{t}$

Decreases exponentially with time t and satisfies the boundary conditions: $U(0,t) = U(l,t) = 0, t \geq 0$. Obtain the solution to this equation $U(x,t)$ at any given time t subject to the above boundary conditions. (10 Marks)

QUESTION THREE (20 MARKS)

The temperature θ in the semi-infinite rod $0 < x < \infty$ is determined by the equation

$$\frac{d\theta}{dt} = K\frac{\partial^2 \theta}{\partial x} \text{ and the conditions}$$

i. $\theta = 0 \; when \; t = 0\,; x > 0$

ii. $\theta = \theta_0$ a constant, when $x = 0, \quad t > 0$

a) By making use of the Sine Fourier transform, transform this equation into a first order ordinary differential equation solvable by an integrating factor. (10 Marks)

hence

b) Solve the resulting equation by a suitable integrating factor. (7 Marks)

c) Use the Fourier inversion formula to show that its temperature distribution at any given time is given by $\theta(x,t) = \frac{2}{\pi}\theta_0 \int_0^\infty \frac{sin\zeta x}{\zeta}\left(1 - e^{-k\zeta^{2}t}\right)d\zeta$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) State the residue theorem (2 Marks)

b) Use the residue theorem to obtain the residues for the integral $I = \int_0^\infty \frac{x^2 dx}{(x^2+16)(x^2+9)^2}$

By considering the upper half plane $|z| \leq R$. (14 Marks)

Hence

c) Evaluate the integral (4 Marks)