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UNIVERSITY EXAMINATIONS 2023/2024

SECOND YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND PHYSICS

SPH 3202: MATHEMATICAL PHYSICS I

DATE: DECEMBER 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Prove the following entities

i. $\vec{\nabla} \times \overrightarrow{\nabla\emptyset} = 0$ (5 Marks)

ii. $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{A}\right) = 0$ (5 Marks)

b) Use the results in (a) above to show that Maxwell's equations of electrodynamics.

$\vec{\nabla} \cdot \vec{E} = 0, \vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{H} = \frac{\partial \vec{E}}{\partial t}$ satisfy the equation $\frac{\partial^2 \vec{H}}{\partial t^2} = \nabla^2 \vec{H}$ (4 Marks)

c) Evaluate the work done $\int \vec{F} \cdot d\vec{s}$ by a fine $\vec{F}$ along a circle $x^2 + y^2 = 4$ in the $x - y$ plane from point A(2,0,0) to B $\left(\sqrt{2}, \sqrt{2,0}\right)$ given that $\vec{F} = \frac{-1}{x}(y + z)$ (10 Marks)

d) Use Demoivie's theorem to find the smallest angle θ for which $(cos\theta + isin\theta)15 = -i$

(6 Marks)

QUESTION TWO (20 MARKS)

a) Consider the time-invariant schrodrigen equations of the form. $-\frac{\hbar^2}{2m}u"(x)+\frac{1}{2}m\omega^2x^2u(x)=Eu(x)$ where $E=$ constant.

i. Use transformation of the form $y=\sqrt{\frac{m\omega}{h}}x$ to transform their equation to the firm

$V"(y)+(\varepsilon-y^2)v(y)=0$ where $\varepsilon=\frac{2E}{hw}=$ constant. (3 Marks)

ii. Solve the equation (ii) above using a solution of the firm $v(y)=B(y)e^{\frac{-y^2}{2}}$ hence show that it leads to a Hemite polynomial of the degree of the form$B"(y)-2y\,B'(y)+(\varepsilon-1)B(y)=0$ (4 Marks)

iii. Show that the Heimite polynomial in (ii) above can be served by power series or ansatz method given by

$B(y)=\sum_{n=0}^{\infty}Cny^n$ to give a recurrence relation. (6 Marks)

v) by considering the value for large n, write down the expression of the recurrence relation in (iii) above (2 Marks)

QUESTION THREE (20 MARKS)

a) State Green's theorem (2 Marks)

b) Prove Green's theorem in (a) above (10 Marks)

c) Use Green's theorem to evaluate $\int xydx+(y^2+1)dy$ where C is a circle centered at the origin with radius 2. (8 Marks)

QUESTION FOUR (20 MARKS)

Consider a Legendre generating function $g(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} p_n(x)\, t^n$

i. Derive the recurrence reflection by

a) Taking derivative w.r.t.t (10 Marks)

b) Taking derivations w.r.t.x (5 Marks)

ii. Given Bessel generating function.

$g(x,t) = e^{\frac{x}{2t}(t^{2-1})} = \sum_{n=\infty}^{+\infty} JN(x)\, t^2$ obtain the recurrence relation w.r.t x. (5 Marks)