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UNIVERSITY EXAMINATIONS 2021/2022

FOURTH YEAR, SECOND SEMESTER SPECIAL SUPPLEMENTARY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE IN ACTUARIAL

SMS 3453: STOCHASTIC PROCESSES

DATE: JANUARY 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Consider the stochastic matrix. $p = \begin{bmatrix} 1/4 & 3/4 \\ 2/3 & 1/3 \end{bmatrix}$

i. Obtain an expression for p^n (5 Marks)

ii. Hence find $\lim_{n\to\infty} \{p^n$ (4 Marks)

b) Let X be a random variable assuming values 0, 1, 2 …, and let $p(X = j) = p_j$ and $p(X > j) = q_j J = 0, 1, 2, \ldots$ if $p(s)$ is the generating function of the sequence $\{p_j\}$ and Q (s) is the generating function of $\{p_j\}$, show that $Q(s) = \frac{1-p(s)}{1-5} - 1 < 5 < 1$. (6 Marks)

c) i. Define the probability general function. (2 Marks)

ii. Using the probability generally function technique, derive the mean and variance of random variable X. (6 Marks)

d) let $z(t)$ be the population size at time t and $pn(t)$ be the probability that a population is of size n at time t, show that the birth death process is defined by the difference – differential equation.

$p_0'(t) = -\mu_0 p_0(t) + \mu_1 p_1(t),\ n \geq 0$. (4 Marks)

e) define the convolution of a sequence. (3 Marks)

QUESTION TWO (20 MARKS)

a) using partial fraction technique, determine an expression of the sequence generated by the function. $\frac{x+3}{(x+1)^2(x+5)}$ (11 Marks)

b) Let X be a random variable write pgf b (s). find the g.f of;

i. $x + 1$ (3 Marks)

ii. $2x$ (2 Marks)

c) Determine the generating function of the sequence $\{a_k\} = \{\frac{1}{k!}\}$ for k=0,1,2,.. (4 Marks)

QUESTION THREE (20 MARKS)

a) Consider the transition probability matrix given in canonical form below.

$$p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.6 & 0.1 & 0 \\ 0.2 & 0 & 0.2 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.2 & 0.4 \end{bmatrix}$$

i. Obtain $\lim_{n\to\infty} p^n$ at exists. (4 Marks)

ii. For each transient state in the chain, obtain $\mu_{,,}$ the recurrence time. (6 Marks)

iii. Obtain for $i \neq j$ μ_{ii} if I, j are in transient class. (6 Marks)

b) In a Markou chain, when is a state I said to be;

i. Recurrent (1 Mark)

ii. Transient (1 Mark)

iii. Ergodien (1 Mark)

iv. Absorbing (1 Mark)

QUESTION FOUR (20 MARKS)

a) Determine the generating function of the sequence. $\{1^2, 2^2, 3^2, 4^2, \ldots.\}$. (8 Marks)

b) Suppose X has a binomial distribution with parameters n, p. Determine the probability generating function of X and hence find the mean and variance of X. (8 Marks)

c) Show that the sum of two poisson random variables with parameter λ_1 and λ_2 is a poisson variable with parameter. $\lambda_1 + \lambda_2$. (4 Marks)

QUESTION FIVE (20 MARKS)

a) i. Define the branching process. (2 Marks)

iii. Find the distribution of $S_N = X_1 + X_2 + \cdots + X_n$ where X_{ii} are independent random variables from a binomial distribution with parameters $n_i$and p for $i = 1,2, \dots . N$. (7 Marks)

b) Prove that for arbitrary a, b and integral K.

$$\sum_{j=0}^{k} \binom{a+k-j-1}{K-j} \binom{b+j-1}{j} = \binom{a+b+k-1}{K}$$

Hint apply $\binom{-a}{K} = (-1)^K \binom{a+k-1}{K}$ (11 Marks)