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University Examinations 2022/2023

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE MATHEMATICS, BACHELOR OF EDUCATION TECHNOLOGY IN ELECTRICAL AND ELECTRONIC ENGINEERING, BACHELOR OF EDUCATION TECHNOLOGY IN MECHANICAL ENGINEERING, BACHELOR OF EDUCATION TECHNOLOGY IN CIVIL ENGINEERING, BACHELOR OF EDUCATION SCIENCE, BACHELOR OF EDUCATION ARTS (MATHS/PE) AND BACHELOR OF EDUCATION ARTS (MATHS/GEOGRAPHY)

SMS 3250: PROBABILITY AND STATISTICS III

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) The continuous random variables have joint distribution $f(x, y)$ and respective marginal density functions $f_1(x)$ and $f_2(y)$. Show that $E[E(y|x)] = E(y)$ (5 marks)

b) x and y are discrete random variables having joint probability mass function given by

$$f(xy) = \begin{cases} \frac{1}{54}(x+y), & x = 1,2,3\ y = 1,2,3,4 \\ 0 & otherwise \end{cases}$$

Determine:

i. Marginal density functions of x and y respectively. Are X and Y independent? (5 marks)

ii. $F(2,3)$ (2 marks)

iii. $Var(Y|X = 2$ (5 marks)

c) Let X and Y be two jointly continuous random variables with joint PDF

$$F(X,Y) = \begin{cases} 2, & x > 0, y > 0, x + y \leq 1 \\ 0, & otherwise \end{cases}$$

Find $COV(x, y)$ (7 marks)

d) Let $Y_1 < Y_2 < Y_3$ be the order statistics of a random sample of size 3 from a distribution having a p.d.f:

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

Compute the joint pdf of $Z = Y_3 - Y_1$ (6 marks)

QUESTION TWO (20 MARKS)

a) Consider the following joint probability mass function

		X		
Y		1	2	3
	4	0.15	0.10	0.05
	3	0.02	0.10	0.05
	2	0.02	0.03	0.20
	1	0.01	0.02	0.25

Obtain the following

i) $f_1(x|y = 2)$ (3 marks)

ii) $E(y|x = 1)$ (5 marks)

iii) $Var(y|x = 1)$ (3 marks)

b) Consider the joint probability density function

$$f(xy)=\begin{cases} x+y & 0<x<1, 0<y<1 \\ 0, & otherwise \end{cases}$$

i. Find $f(y|X=x)$ (5 marks)

ii. Compute $E(y|X=x)$ (4 marks)

QUESTION THREE (20 MARKS)

a) Consider the following joint pdf of x and y

$$f(xy)=\begin{cases} c(2x+3x) & x=0,1, y=0,1,2 \\ 0, & otherwise \end{cases}$$

i. Compute the value of the constant c (2 marks)

ii. Find the joint moment generating function of x and y (5 marks)

iii. Use (ii) to determine the marginal moment generating functions and hence show that x and y are dependent (5 marks)

iv. Compute $E(x+3)$ (3 marks)

b) A fair dice is rolled n times. Let X be the number of 1's that are observed Y be the number of 2's that are observed. Find $COV(x,y)$ *and* $p(x,y)$ (5 marks)

QUESTION FOUR (20 MARKS)

a) The continuous random variable x has a uniform distribution over the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Show that the p.d.f of $Y=\tan x$ is given by (7 marks)

$$g(y)=\begin{cases} \dfrac{1}{\pi(1+y)^2} & -\infty<y<\infty \\ 0, & otherwise \end{cases}$$

b) Define the F-distribution. The random variable Y has a F-distribution with m and n degrees of freedom. Derive the mean of the random variable Y (10 marks)

c) Suppose that the random vector $\underline{X}=(x_1,x_2,x_3)'$ has the joint moment generating function

$$M\left(t_1,t_2,t_3\right)=\exp\left\{\frac{1}{2}\left(2t_1^2+3t_2^2+4t_3^2+2t_1t_2+2t_1t_3+2t_2t_3\right)\right\}$$

Determine the marginal m.g.f of

i. X_1 (1 mark)

ii. $\left(X_1X_2\right)$ (2 marks)

QUESTION FIVE (20 MARKS)

a) Given x and y are independent standard normal variates find the joint p.d.f of $u=x+y$ and $v=y-x$. Use the moment generating function technique (8 marks)

b) Define the term order statistics (3 marks)

i. Let $y;y_1<y_2<y_3<y_4<y_5$ denote the order statistics of a random sample of size $n=5$ from a distribution having p.d.f.

$$f\left(x\right)=\begin{cases} e^{-x} & ,0<x<\infty \\ 0 & , \quad otherwise \end{cases}$$

Find the p.d.f. of the minimum y_1 and the median y_3 (9 marks)