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University Examinations 2022/2023

SECOND YEAR, FIRST SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE
DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3214: ACTUARIAL MATHEMATICS 1

DATE: AUGUST 2023

TIME: 2 HOURS

INSTRUCTIONS: Answer question one and any other two questions

QUESTION ONE (30 MARKS)

- a) Differentiate between constant force of mortality and uniform distribution of deaths (2 marks)
- b) Calculate $0.5p_{90.25}$ using the method of uniform distribution of deaths
Basis: Mortality ELT15 (Males) (4 marks)
- c) Show that $\bar{A}_{X:n|} = 1 - \delta \bar{a}_{X:n|}$ (5 marks)
Basis:
Force of mortality μ_x is constant for all x
Force of interest δ throughout
- d) Evaluate $A_{[50]}$ and $\bar{A}_{[50]}$ based on AM92 Select mortality at 6% pa interest (3 marks)
- e) (i) describe how education may affect mortality (2 marks)
(ii) List three examples of the influence of education on mortality (3 marks)
- f) An individual, aged 32, buys a deferred annuity that is payable from age 60 for 5 years certain and for life thereafter. The amount of the annuity is \$600 pa, payable annually in arrears. Calculate the annual premium payable in advance for this annuity (8 marks)
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Basis:

Mortality: AM92 Ultimate Mortality

Rate of interest: 6% pa

g) In a certain population, the force of mortality equals 0.025 at all ages. Calculate:

- (i) The probability that a new-born baby will survive to age 5 (1 mark)
- (ii) The probability that a life aged exactly 10 will die before age 12 (2 marks)

QUESTION TWO (20 MARKS)

a) Calculate ${}_4p_{63.5}$ based on PFA92C20 mortality using:

- (i) The constant Force of Mortality (CFM) assumption (5 marks)
- (ii) The uniform Distribution of Deaths (UDD) assumption (5 marks)

b) Consider the ELT15(males). The following is an extract from that table

Age, x	l_x	d_x	q_x	μ_x
0	100,000	814	0.00814	
1	99,186	62	0.00062	0.00080
2	99,124	38	0.00038	0.00043
3	99,086	30	0.00030	0.00033
4	99,056	24	0.00024	0.00027
5	99,032	22	0.00022	0.00023
6	99,010	20	0.00020	0.00021
7	98,953	18	0.00018	0.00018

Use the above extract to calculate the values of:

- (i) p_3 (2 marks)
- (ii) ${}_3p_2$ (1 mark)
- (iii) ${}_4q_1$ (2 marks)
- (iv) l_8 (1 mark)

c) Show that $E[T_X] = \int_0^\infty {}_t p_x \mu_x + t dt = \int_0^\infty {}_t p_x dt$ (5 marks)



QUESTION THREE (20 MARKS)

- a) Calculate the expectation of the present value of the benefits from each of the following contracts issued to a life aged exactly 40, assuming that the annual effective interest rate is 4% and AM92 select mortality applies
- (i) A deferred whole life annuity-due with a deferred period of 20 years, under which payments of £15,000 are made in advance while the policy holder is alive after the deferred period has elapsed (5 marks)
 - (ii) A guaranteed annuity with a guarantee period of 20 years, under which payments of £8,000 are made annually in arrears for a minimum of 20 years and for life thereafter (5 marks)
- b) (i) let W be a random variable representing the present value of the benefits payable under an immediate life annuity that pays 1 per year in advance, issued to a life aged x . Find the expression of the variance of W in actuarial notations and specify the appropriate rate of interest of the assurances (6 marks)
- (ii) A life office issues such a policy to a life aged exactly 55. The benefit is \$300 per annum. Calculate the standard deviation of the annuity (4 marks)
- Basis:
Mortality: AM92 Select Mortality
Rate of interest: 6% per annum throughout

QUESTION FOUR (20 MARKS)

- a) A life insurance company sells 20-year with-profit endowment assurances to lives aged 40 exact. The basic sum assured is \$50,000 and compound bonuses of 1.923% are added to the sum assured at the end of each year, provided the policy holder survives to the end of the year. The death benefit is payable at the end of the year of death. Level premiums are payable monthly in advance
- Basis:
Mortality AM92 Select
Interest 6% per annum



Expenses

Initial	50% of the first year's premiums, payable at the start of the contract
Renewal	5% of all premiums, including the first year's premiums, payable at the start of each year

- (i) Show that the monthly premium is \$182.64 (12 marks)
- (ii) Find the gross premium prospective reserve just before the start of the fifteenth year of the policy, assuming that bonuses have been declared according to the initial assumptions (8 marks)

QUESTION FIVE (20 MARKS)

- a) An annuity is payable continuously throughout the lifetime of a person now aged exactly 60, but for at most 10 years. The rate of payment at all times t during the first 5 years is 5,000pa and thereafter it is 7,000 pa.

The force of mortality of this life is 0.03 pa between the ages of 60 and 65 and 0.04 pa between the ages of 65 and 70

Calculate the expected present value of this annuity assuming a force of interest of 0.05 pa (10 marks)

- b) (i) Calculate $\bar{A}_{30:25|}$ and $\bar{a}_{30:25|}$ independently, assuming AM92 mortality and 6% pa interest (6 marks)
- (ii) State the assumptions underlying the approximations on which your values calculated in part (i) are based (2 marks)
- (iii) Hence verify the usual premium conversion relationship holds approximately between these two functions (2 marks)

