



MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya

Tel: +254(0) 799 529 958, +254(0) 799 529 959, + 254 (0) 712 524 293,

Website: info@must.ac.ke Email: info@must.ac.ke

University Examinations 2023/2024

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER, BACHELOR OF EDUCATION ARTS, BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF EDUCATION SCIENCE

FOURTH YEAR FIRST SEMESTER BACHELOR OF EDUCATION TECHNOLOGY IN MECHANICAL ENGINEERING, BACHELOR OF EDUCATION IN ELECTRICAL ENGINEERING, BACHELOR OF CIVIL ENGINEERING

SMS 3200: PROBABILITY AND STATISTICS II

DATE: DECEMBER 2023

TIME: 2 HOURS

INSTRUCTIONS: Answer question *one* and any other *two* questions

QUESTION ONE (30 MARKS)

- a) Explain the difference between expectation and variance of a discrete random variable (2 marks)
- b) Outline four properties of a cumulative frequency distribution function (4 marks)
- c) Given $f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$
- i. Show that it is a *pdf* (2 marks)
- ii. Find the moment generating function (3 marks)
- iii. Hence find the mean and the variance of x (4 marks)

d) A random variable x has pmf $f(x) = \begin{cases} \frac{x}{6}, & x = 1, 2, 3 \\ 0, & elsewhere \end{cases}$

Find the cdf of the random variable and sketch the graph (5 marks)

e) An arrival process follows a Poisson distribution with mean of 4 arrivals per time unit.

What is the probability that an arrival occurs after 33 time units (5 marks)

f) Jane has just begun her new job as on the sales force of a very competitive company. In a sample of 16 sales calls it was found that she closed the contract for an average value of 108 dollars with a standard deviation of 12 dollars. Test at 5% significance that the population mean is at least 100 dollars against the alternative that it is less than 100 dollars.

Company policy requires that new members of the sales force must exceed an average of 100 per contract during the trial employment period. Can we conclude that Jane has met this requirement at the significance level of 5%? (5 marks)

QUESTION TWO (20 MARKS)

a) A random variable X has a pmf given by;

$$f(x) = \begin{cases} p(1-p)^x, & x = 0, 1, 2, \dots \\ 0, & elsewhere \\ p = \frac{1}{4} \end{cases}$$

Find the:

i. Moment generating function (mgf) (5 marks)

ii. Use the mgf to compute the mean and the variance (5 marks)

b) Given the following probability density function of a continuous random variable, find the mode and median of the distribution

$$f(x) = \begin{cases} -x^2 + 2x - \frac{1}{6}, & 0 < x < 2 \\ 0, & elsewhere \end{cases} \quad (10 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) The average life span of a dishwasher is normally distributed with a mean of 9 years and a standard deviation of 1.8years. What is the probability that a dishwasher will last
- More than 10 years (4 marks)
 - Fewer than 7 years (4 marks)
 - Between 6 and 7 years (4 marks)
- b) A multiple choice test contains 25 questions each with 4 answers. Assume that a student guesses each question. What is the probability that the student answers
- More than 20 questions correctly (4 marks)
 - Less than 5 questions correctly (4 marks)

QUESTION FOUR (20 MARKS)

- a) Given the gamma density function

$$f(x) = \begin{cases} x^{\alpha-1} e^{-\frac{x}{\beta}} \\ \Gamma \alpha \beta^{\alpha}, & \alpha x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Determine the mgf and hence the mean and variance (10 marks)

- b) Consider a continuous random variable with the following probability density function $f(x) = 3x^2$ for $0 < x < 1$. Use the change-of- variable technique to find the probability density function of $Y = x^2$ (5 marks)
- c) A manufacturer of salad dressings uses machines to dispense liquid ingredients into bottles that move along a filling line. The machine that dispenses salad dressings is working properly when 8 ounces are dispensed. Suppose that the average amount dispensed in a particular sample of 35 bottles is 7.91 ounces with a variance of 0.03 ounces squared². Is there evidence that the machine should be stopped and production wait for repairs? The lost production from a shutdown is potentially so great that management feels that the level of significance in the analysis should be 99% (5 marks)

QUESTION FIVE (20 MARKS)

- a) The lengths of telephone conversations, in minutes, by sales reps of a certain company are modelled by the continuous random variable T . The probability density function of T is denoted by $f(t)$ and is given by

$$f(t) = \begin{cases} kt, & 0 < t < 12 \\ 0, & \text{otherwise} \end{cases}$$

- i. Find K (2 marks)
 - ii. Determine $P(T > 5)$ (2 marks)
 - iii. Show by calculation $E(T) = \text{Var}(T)$ (6 marks)
- b) Use the gamma function to show that $\Gamma\frac{1}{2} = \sqrt{\pi}$ (5 marks)
- c) A fair coin is tossed twice. Let X be the number of heads that are observed. Construct the probability distribution of X and find the probability that at least one head is observed (5 marks)