MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.
Tel: +254(0) 799 529 958, +254(0) 799 529 959, +254 (0)712 524 293
Website: www.must.ac.ke Email: info@mucst.ac.ke

UNIVERSITY EXAMINATIONS 2022/2023

FIRST YEAR, SECOND SEMESTER EXAMINATION FOR DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS

SMA 8110: ADVANCED NUMERICAL ANALYSIS

DATE: DECEMBER 2022 **TIME: 3 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Discuss three main properties of a discretization scheme. (3 Marks)

b) In reference to a numerical solution algorithm, explain the meaning of the following terms

i. Convergence

ii. Consistence

iii. Stability

c) In a computation of a steady compressible laminar flow, it was found out that the computed results do not agree with experimental results. Give three possible reasons for the variance. (3 Marks)

d) Write down.

i. Upwind

ii. Central difference approximations of the convective flux, $\rho u\phi$, of the scalar quantity ϕ, transported by a fluid a density ρ and local velocity u, at point mid-way between two nodes. (4 Marks)

e) The unsteady $1-D$, equation

$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=\Gamma\frac{\partial^2 T}{\partial x^2}$,where u *and* T are constants, is approximated at $[x_i, t_n]$ by

$\frac{T_i^{n+1} - T_i^n}{\Delta t} + \frac{\mu}{2\Delta x}\left[T_{i+1}^n - T_{i-1}^n\right] = \frac{\Gamma}{\Delta x^2}\left[T_{1+1}^n - 2T_i^n + T_{n-1}^n\right]$ where $T_i^n = T(x_i, t_n)$ and Δx is the mesh spacing.

Use Taylor series method to determine the truncation errors in space and time associated with the approximation. (8 Marks)

f) The distribution of a scalar quantity ϕ is described by the following conservation quantity.

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = 0$$

Where u *and* v, the components of velocity in x any y direction respectively, are constant and positive.

i. What is the main advantage of the finite-volume technique over the finite-difference method? (2 Marks)

ii. Derive a finite-volume approximation of the equation using a control volume with equal spacing. (7 Marks)

QUESTION TWO (20 MARKS)

Discretization of many equations requires approximations of the first derivatives $\frac{\partial f}{\partial x}$ and second derivative term $\frac{\partial^2 f}{\partial x^2}$.

i) Using appropriate diagrams, explain the forward, backward and central difference approximations of $\frac{\partial f}{\partial x}$ and comment on the accuracy for each approximation (6 Marks)

ii) Consider a uniform grid with grid spacing $\Delta x.$ using the central difference scheme, derive the second order finite difference formulation for $\frac{\partial f}{\partial x}$ and $\frac{\partial^2 f}{\partial x^2}$ from Taylor's series expansion (8 Marks)

iii) Derive the fourth-order central-difference formula for $\frac{\partial f}{\partial x}$ (6 Marks)

QUESTION THREE (20 MARKS)

a) The finite volume method generates approximations that satisfy any conservation property of the original partial differential equation regardless of the mesh spacing. Explain (5 Marks)

b) Use the finite volume method in conjunction with mesh arrangement shown in the figure below to obtain an approximation to $\frac{\partial^2 \phi}{\partial x \partial y}$ integrated over the shaded area shown in the figure. (15 Marks)

QUESTION FOUR (20 MARKS)

a) Simple partial differential equations can be classified in three; Elliptic, Parabolic and hyperbolic. Describe giving examples, the main features of each type their traditional solution methods. (5 Marks)

b) Two dimensional heat conduction can be described by the Poisson equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = f(x, y)$$

Where T is the temperature a $f(x, y)$ is the source term. We are interested in the steady state solution in the unit square $0 \le x \le 1, \quad 0 \le y \le 1$

(i) Discretise the Poisson equation using CDS for second order derivative and derive the finite difference equation. (8 Marks)

(ii) Explain how to solve the finite difference equation using ADI (7 Marks)

QUESTION FIVE (20 MARKS)

The equation describing unsteady heat conduction in a two-dimensional system can be expressed as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k \frac{\partial T}{\partial y}\right] + S_T$$

(i) Simplify the equation for a case of constant fluid properties and without any additional heat source (5 Marks)

(ii) Discretize the simplified equation in space and find by applying the finite-volume method on the control volume arrangement shown in the figure below (15 Marks)

You may use the time integral of T at node P, approximated by

$$\int_{t}^{t+\Delta t} T_p dt = \left[\theta T_p^{New} + [1-\theta]T_p^{old}\right] \Delta t$$