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University Examinations 2023

FIRST YEAR, FIRST SEMESTER EXAMINATION THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

SMA 5002: FUNCTIONAL ANALYSIS 1

DATE: APRIL 2023 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer any three questions*

QUESTION ONE (20 MARKS)

a) (i) Define a metric in a metric space (X, d) (4 marks)

(ii) Show that $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ for $x = (x_1, x_2)$ and $y = (y_1, y_2) \in \mathbb{R}^2$ is a metric (4 marks)

b) (i) Define orthogonality of two elements in an inner product space (2 marks)

(ii) In an inner product space, the corresponding norm satisfies the Schwarz inequality $|< x, y >| \leq \|x\| \, \|y\|$ prove the inequality (10 marks)

QUESTION TWO (20 MARKS)

a) (i) Define what is meant by a linear space over k- the field of scalars (4 marks)

(ii) Distinguish between a Hamel base and Schadder base for a linear space X (3 marks)

b) Define the following:

i) A linear operator (2 marks)

ii) A normal linear space (4 marks)

c) Prove that $T \in \beta\,(X, Y)$ where X and Y are normal linear spaces if and only if T is continuous (7 marks)

QUESTION THREE (20 MARKS)

a) Define the following

   (i) A Cauchy sequence (2 marks)

   (ii) A complete normed linear space (2 marks)

b) Prove that a Cauchy sequence of elements of X, where X is infinite dimensional normal space need not converge (5 marks)

c) Prove that the dual space of ι^p $(1 < P < \infty)$ is ι_q $(1 < q < \infty)$, $\frac{1}{P} + \frac{1}{Q} = 1$ (6 marks)

d) Prove that strong convergence is not equivalent to weak convergence (5 marks)

QUESTION FOUR (20 MARKS)

a) Define what is meant by saying that a normal linear space X is isometrically isomorphic to Y. (3 marks)

b) Let X be a normal linear space prove that X is isometrically isomorphic to a linear subspace $\hat{X}$ of X^{**} (10 marks)

c) Suppose that μ is a linear subspace of a normal linear space X. If $F(x) = f(x)$ on M and $\|F\| = \|f\|$ (7 marks)

QUESTION FIVE (15 MARKS)

a) State and prove uniform boundedness theorem (8 marks)

b) Explain clearly the differences or similarities if any between the open mapping theorem and closed graph theorem (6 marks)

c) Let $x = [0, 1]$, $Y = \mathbb{R}$ and $T: X \rightarrow \mathbb{R}$ be define by $Tx = x^2$, $x \in [0,1]$. The graph of T is given by : $G(T) = \{(x, Tx): x \in [0, 1]\}$ show that the graph T, G9T) is closed (6 marks)