MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY
P.O. Box 972-60200 – Meru-Kenya.
Tel: +254 (0)799529958, +254 (0)799529959, +254 (0)712524293
Website: www.must.ac.ke Email: info@must.ac.ke

University Examinations 2023

FIRST YEAR, FIRST SEMESTER EXAMINATION THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

SMA 5001: MEASURE AND INTEGRATION

DATE: APRIL 2023 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) (i) Define a sigma algebra of a set X (2 marks)

(ii) Prove that a sigma algebra in closed with respect to taking countable intersections (3 marks)

(iii) Determine whether the collection M of subsets of X are sigma algebras in $X = \{1, 2, 3, 4, 5\}$

$$M = \{\phi, \ \{1\}, \{2\}, \{3, 4\}, \ \{1, 2\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$$ (3 marks)

(iv) Let $X = \{1, 2, 3, 4\}$ and $\beta = \{\{1\}, \{2, 4\}\}$. Find the sigma – algebra generated by β

(3 marks)

b) If f and g are integrable functions and c is a real constant, prove that;

(i) $\int (c\, f)\, d\, \mu \ = c \int f\, d\mu$ (4 marks)

(ii) $\int (f + g)\, d\mu \ = \ \int f\, d\mu + \ \int g\, d\mu$ (4 marks)

c) Give a counter example to show that if f^2 and $|f|$ are measurable, f is not necessarily measurable (5 marks)

d) State fatous lemma and give an example to show that the inequality cannot be replace with equality (6 marks)

QUESTION TWO (15 MARKS)

a) If $(X, \ x, \ \mu)$ is a measure space and if $\{f_n\}$ is a sequence of measurable functions on X, prove that $\{x: \lim f_n \ (x) \ exists\}$ is a measurable set (9 marks)

b) Given that E_1 and E_2 are two measurable sets, prove that $E_1 \ \cup E_2$ is measurable (6 marks)

QUESTION THREE (15 MARKS)

a) Suppose that $E \ C\mathbb{R}$ has a finite lebergue measure. Prove that $\mu \ (E \ \cap \ [x, \infty]) \rightarrow 0$ as $x \ \rightarrow \ \infty$ (6 marks)

b) Prove that if f is measurable and $f > 0$ then the truncation function f_A defined by

$$f_A \ (x) = \begin{cases} f(x) & , \ if \ \ |f \ (x)| \ \leq A \\ A & , \ if \ \ f \ (x) \ > A \\ -A & , \ if \ \ f(x) \ < A \end{cases}$$

Is measurable (9 marks)

QUESTION FOUR (15 MARKS)

a) Explain what is meant by "a proposition that holds almost everywhere." (2 marks)

b) If $\mu \ (E) = 0$, prove that E is measurable (5 marks)

c) Prove that E is measurable $iff \ E^c$ is measurable (8 marks)

QUESTION FIVE (15 MARKS)

a) Let $f: x \ \rightarrow Y$ be a function. If y is a sigma – algebra of subsets of Y, prove that the class $f^{-1}(E): E \in Y$ is a sigma – algebra of subset of X (8 marks)

b) Given that $f: x \rightarrow \ \mathbb{R}_e$ is measurable, prove that $\{x \ \in X : f(x) = c, \ c \ \in \mathbb{R}_e\}$ is measurable (7 marks)