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UNIVERSITY EXAMINATIONS 2022/2023

FOURTH YEAR, SECOND SEMESTER SPECIAL SUPPLEMENTARY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 3457: ANALYTICAL APPLIED MATHEMATICS II

DATE: JANUARY 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) State the transformation properties of the tensors A_i and B^{ij}. Hence obtain the transformation properties of the tensor $A_i B^{ij}$ (5 Marks)

b) Classify the integral equation

$$x^2 = \phi(x) - \int_0^1 (x^2 - y^2)\phi(y)dy$$ (4 Marks)

c) Find the convolution $f * g$ when $f(t) = t^2$ and $g(t) = t^3$. (4 Marks)

d) Write down the transformation law for the tensor A^{pq}_{rst} (2 Marks)

e) Convert the differential equation $y''(x) + y(x) = 0$ with conditions $y(0) = 1, y^1(0) = 0$ to an integral equation. (5 Marks)

f) Prove the recurrent relation $\frac{d}{dx}\left\{x^{-n} I_n^{(x)}\right\} = x^{-n} I_{n+1}^{(x)}$ for the modified Bessel function.

(5 Marks)

g) Find the first two terms of the Neumann series for the integral equation

$$u(x) = 1 + \in \int_0^1 (x - y)u(y)dy$$ (5 Marks)

QUESTION TWO (20 MARKS)

a) Write down the tensor $\bar{A}_{K^{ij}}$ in full. (2 Marks)

b) Show that the contract of the outer product of the tensors A^p and B_q is an invariant.

(5 Marks)

c) Prove the relation $\frac{d}{dx}\left\{x^{-n}k_n{}^{(x)}\right\} = x^{-n}k_{n+1}{}^{(x)}$ for the modified Bessel equation. (6 Marks)

d) Show that the function $y = (x+1)^2$ is a solution of the volterra equation.

$y(x) = e^{-x} + 2x + \int_0^x e^{t-x} y(t) dt$ (7 Marks)

QUESTION THREE (20 MARKS)

a) Solve the integral equation

$y(t) = t - \int_0^t \sin\,(t-\beta)\, y(\beta)\, d\beta$ using Laplace transforms. (16 Marks)

b) Write down the general form of:

i. Fredholm integral equation of first and second kind. (2 Marks)

ii. Volterra integral equation of first and second kind. (2 Marks)

QUESTION FOUR (20 MARKS)

a) Find the inverse Laplace transform for the function $\frac{1}{S(S^2+1)}$ by convolution. (7 Marks)

b) Derive an equivalent volterra integral equation for the initial value problem

$y'' + 5y' - 6y = 0$ given that $y(0) = 1$ and $y'(0) = 1$ (8 Marks)

c) Prove the recurrence relation

$\frac{d}{dx}\left\{x^{-n} I_n{}^{(x)}\right\} = x^{-n} I_{n+1}{}^{(x)}$ (5 Marks)

QUESTION FIVE (20 MARKS)

a) Use the generating function for the Bessel functions to prove that $J_n^{(x+y)} = \sum_{r=-\infty}^{\infty} Jr^{(x)} J_{n-r}^{(y)}$.

(8 Marks)

b) Given the generating function for the Bessel functions. $\exp\left\{\frac{x}{2}\left(t+\frac{1}{t}\right)\right\} = \sum_{n=-\infty}^{\infty} I_n^{(x)} t^n$, show that the coefficient of t^n is $I_n(x)$ (12 Marks)