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University Examinations 2022/2023

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE STATISTICS, BACHELOR OF SCIENCE MATHEMATICS AND BACHELOR OF ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER BACHELOR OF SCIENCE (MATHS/PHYSICS)

SMA 3254: LINEAR ALGEBRA II

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Show that $T: R^2 \to R^2$ defined by $T(x, y) = (x + y, 3x)$is a linear transformation (4 marks)

b) Let $f = \{f_1, f_2, f_3\}$be a basis for $\mathbb{R}^3$and $\{f_1 = (1,1,1), f_2 = (0,1,1), f_3 = (0,0,1)\}$. Find the coordinate vector of $V = (5,-1,9)$ relative to the basis f (5 marks)

c) Determine whether the following two matrices are similar (4 marks)

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} and\ B \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

d) Let V be the vector space of 2x2 matrices over $\mathbb{R}$ and let $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$,let T be the linear operator on V defined by $T(A) = MA$. Find the trace of T (6 marks)

e) Let T be a linear mapping on $\mathbb{R}^2$ define by $T(x_1 - 3x_2, x_1 + 2x_2)$. Find the matrix of T in the basis $\{e_1 = (1,0), e_2 = (0,1)\}$ (4 marks)

f) Compute the determinant of $A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{bmatrix}$ (6 marks)

QUESTION TWO (20 MARKS)

a) Let $T: R^3 \rightarrow R^2$be the linear mapping defined by $T(1,1,1) = (1,0), T(1,1,0) = (2,-1)$ and $T(1,0,0) = (4,3)$. Find $T(x, y, z)$then compute $T(2,-3,5)$ (8 marks)

b) Find the characteristic polynomial, minimal polynomial and the eigenvalues of the matrix

$A = \begin{bmatrix} 4 & -5 & 3 \\ 2 & -3 & 2 \\ -1 & 1 & 0 \end{bmatrix}$ (8 marks)

c) Let $T : P_2 \rightarrow P_2$be the linear operator defined by $T(a_0 + a_1 x + a_2 x^2) = a_0 + a_1(3x - 5) + a_2(3x - 5)^2$. Find the matrix for $[T]_\beta$ with respect to the standard basis $(1, x, x^2)$ $for\ p_2$ (4 marks)

QUESTION THREE (20 MARKS)

a) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$be defined by $T(x, y) = (x + y, -2x + 4y)$. Find the determinant of T

(5 marks)

b) Consider the basis $\{f_1 = (1,2), f_2 = (2,3)\ and\ g_1 = (1,3), g_2 = (2,5)\}$two basis of $\mathbb{R}^2$

i. Find the transition matrix P from $\{f_i\}$to $\{g_i\}$and the transition matrix Q from $\{g_i\}$to $\{f_i\}$ (7 marks)

ii. Verify that $Q = P^{-1}$ (1 mark)

iii. Show that $P^{-1}[V]_f = [V]_g$ (3 marks)

c) Verify the Carley-Hamilton Theorem for the matrix $P = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Compute the determinant of $\begin{bmatrix} 4 & 0 & -7 & 3 & 5 \\ 0 & 0 & -2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{bmatrix}$ (7 marks)

b) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$

i. Find the matrix representation of T relative to the following basis of $\mathbb{R}^3$and $\mathbb{R}^2$

$(f_1 = (1,1,1), f_1 = (1,1,0), f_1 = (1,0,0))\, and\, (e_1 = (1,3), e_2 = (2,5))$ (5 marks)

ii. Verify that for any vector$v \epsilon \mathbb{R}^3$, $[T]_f^e [v]_f = [T(v)]_e$ (8 marks)

QUESTION FIVE (20 MARKS)

a) The set $\{1, t, \sin 3t, \cos 3t\}$is a basis of a vector space V of $f: \mathbb{R} \to \mathbb{R}$. Let D be the differential operator on V, that is, $D(f) = \frac{df}{dt}$. Find the matrix of D in the given basis

(4 marks)

b) Diagonalize $B = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix}$ (12 marks)

c) Show that $T: M_{22} \to \mathbb{R}^3$defined by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b, c + d, 2)$ is not a linear transformation (4 marks)