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University Examinations 2022/2023

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN EDUCATION SCIENCE
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF TECHNOLOGY IN ELECTRICAL ENGINEERING
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING
SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF TECHNOLOGY OF CIVIL ENGINEERING

SMA 3250: VECTOR ANALYSIS

DATE: AUGUST 2023 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question one and any other two questions*

QUESTION ONE (30 MARKS)

a) Distinguish between a vector and a scalar (2 marks)

b) Given the vectors $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{B} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{C} = 4\hat{\imath} + 3\hat{k}$. Find the magnitude of $3\vec{A} + 2\vec{B} - \vec{C}$ (4 marks)

c) Determine the value of P such that $\vec{A} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ and $\vec{B} = 2\hat{\imath} + P\hat{\jmath} + \hat{k}$ are perpendicular (3 marks)

d) Find the angle between $\vec{P}$ and $\vec{Q}$ given $\vec{P} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $Q = \hat{\imath} - 3\hat{\jmath} + \hat{k}$ (4 marks)

e) If A and B are differentiable functions of scalar u, prove (4 marks)

$$\frac{d}{du}\left(\vec{A}.\vec{B}\right) = \vec{A}.\frac{d\vec{B}}{du} + \frac{d\vec{A}}{du}\cdot\vec{B}$$

f) If $\vec{A} = 2t\hat{\imath} + 5t^2\hat{\jmath} - 6\hat{k}$ and $\vec{B} = cost\ \hat{\imath} + \sin t\ \hat{\jmath}$, find $\frac{d}{dt}\left(\vec{A}\ x\ \vec{B}\right)$ (5 marks)

g) If $\vec{R}(u) = (u - u^3)\ \vec{\imath} + 2u\vec{\jmath} + 5u\vec{k}$. (3 marks)

Find

$$\int_1^3 \vec{R}\,(u)du$$

h) If $\emptyset(x, y, z) = 2xy^2 + x^3z^3$, find $\nabla\ \emptyset$ at the point (-1, 2, -1) (5 marks)

QUESTION TWO (20 MARKS)

a) $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $\vec{B} = 5\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ compute (6 marks)

(i) $\vec{A}\ x\ \vec{B}$

(ii) $prod_{\vec{A}}\vec{B}$

b) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx = x^2dy$ where c is closed curve of the region bounded by $y = x$ and $y = x^2$ (14 marks)

QUESTION THREE (20 MARKS)

a) If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, evaluate $\int_c \vec{A}.dr$ from (0,0,0) to (1,1,1) along the curve $x = t$, $y = t^2$ and $z = t^3$ (4 marks)

b) Find the total work done in moving a particle in a force given by $\vec{t} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$to $t = 2$ (7 marks)

c) Evaluate $\iint_c A\ nds$, where $A = 18z\vec{\imath} - 12\vec{j} + 3y\vec{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant (9 marks)

QUESTION FOUR (20 MARKS)

a) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$ (8 marks)

b) If $\vec{A} = xz^2\vec{\jmath} - 2x^2yz\vec{\jmath} + z^4\vec{k}$, find curl $\vec{A}$ (6 marks)

c) Find the volume of the parallelepiped with side $\vec{a} = 2\hat{\imath} + 4\hat{k}$, $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Show that the vectors $\vec{A} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{B} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}, \vec{C} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ from a right angled triangle (5 marks)

b) Prove that the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$ is $\left|\vec{A}x\vec{B}\right|$ (4 marks)

c) A particle moves along the curve $x = 2t^2 + t, y = t^2 - 3t, z = 5t - 5$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ (4 marks)

d) Find a function $f(x, y, z)$ so that $\frac{2xy}{z^3}\hat{\imath} + \left(2y + z^2 + \frac{R^2}{Z^3}\right)\hat{J} - \left(4z^3 - 2yz - \frac{3x^2y}{z^4}\right)\hat{k}$ (7 marks)