



MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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University Examinations 2022/2023

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR
THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR
THE DEGREE OF BACHELOR OF SCIENCE IN EDUCATION SCIENCE

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR
THE DEGREE OF BACHELOR OF EDUCATION ARTS

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR
THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SMA 3212: NUMBER THEORY

DATE: AUGUST 2023

TIME: 2 HOURS

INSTRUCTIONS: Answer question one and any other two questions

QUESTION ONE (30 MARKS)

- a) If $a|b$ and $a|c$ and if m and n are integers show that $a|(mb + nc)$ (3 marks)
- b) Write m in the form $m = nq + r$, $0 \leq r \leq |n|$ given that
- (i) $m = -17, n = 5$ (2 marks)
- (ii) $m = -39, n = -8$ (2 marks)
- c) Find the solution to the equation $42x \equiv 50 \pmod{91}$ (4 marks)
- d) State the Euler's Theorem in number theory (2 marks)
- Show that $a^{\phi(n)} \equiv 1 \pmod{n}$ where $n=9$ (4 marks)
- e) Using Euclidean algorithm, calculate the greatest common divisor of 714, 2205 and 2030 (5 marks)



- f) Determine m_1, m_2, \dots, m_6 , given that $m_k = P_1 \cdot P_2 \dots \dots P_k + 1$ where $P_1 = 2, P_2 = 3 \dots$
 Are prime numbers in ascending order (6 marks)
- g) Prove that $0 < |a| \leq |b|$, if $a|b$ and $b \neq 0$ and $a, b \in \mathbb{Z}$ (2 marks)

QUESTION TWO (20 MARKS)

- a) (i) Define a residue class modulo n (3 marks)
 ii) List down residue classes modulo 7 (7 marks)
- b) (i) Define the Pell's equation (3 marks)
 (ii) Show that the following rational approximations to $\sqrt{2}$ ones all "good" in the sense that
 $a^2 - 2b^2 = \pm 1$: $\frac{a}{b} = \frac{1}{2}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}$ (7 marks)

QUESTION THREE (20 MARKS)

- a) Determine whether 17 is a prime number by deciding whether or not $16! \equiv -1 \pmod{17}$ (6 marks)
- b) Show that $18! \equiv -1 \pmod{437}$ (7 marks)
- c) Arrange the integers 2, 3, 4, ..., 21 in pairs (a,b) with the property that $ab \equiv 1 \pmod{23}$ (7 marks)

QUESTION FOUR (20 MARKS)

- a) State and prove the Euclidean theorem on infinity of prime number (10 marks)
- b) (i) $(P - 1)! \equiv -1 \pmod{P}$ if P is prime. Prove (6 marks)
 (ii) Use $P = 1$, to verify the above proof (4 marks)

QUESTION FIVE (20 MARKS)

- a) Calculate $\phi(9702)$ and $\phi(873)$ (6 marks)
 Prove that $\phi(n)$ is even $n > 2$ (6 marks)
- b) Solve the Diophantine equation $621y + 738x - 45 = 0$ (8 marks)

