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University Examinations 2023/2024

SECOND YEAR, FIRST SEMESTER EXAMINATION THE DEGREE OF BACHELOR OF SCIENCE IN ACTURIAL SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE MATHEMATICS

SMA 3211: LINEAR ALGEBRA 1

DATE: DECEMBER 2023 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question one and any other two questions*

QUESTION ONE (30 MARKS)

a) Find the value of y such that $\begin{pmatrix} y & 5 \\ -4 & y-9 \end{pmatrix}$ is singular (3 marks)

b) Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$. Find AA^T (3 marks)

c) Find a, b, c and d if $\begin{pmatrix} a+b & 2c+d \\ a-b & c-d \end{pmatrix} = 2\begin{pmatrix} 3 & 5 \\ 1 & 4 \end{pmatrix}$ (3 marks)

d) Determine m so that the vectors $u = (m^2, -7, 3)$ and $v = (m, m, -2)$ are orthogonal (4 marks)

e) Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$. Find $g(A)$ if $g(x) = x^2 + 3x - 10$ (4 marks)

f) Evaluate the determinant of matrix $A = \begin{pmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{pmatrix}$ (3 marks)

g) Let $T: R^3 \to R^3$ be defined by $T(x, y, z) = (x - y + z, 2x - 3y + z, 2x - 3y + z)$. Find the matrix representation of T relative to the standard basis. (3 marks)

h) Let $T: \Re^2 \to \Re$ be defined by $T(x, y) = (x + y, x)$. Examine if T a linear mapping (4 marks)

i) Find all solution of the system $\begin{matrix} 2r + & 3s + & 4t = 16 \\ 8r + & 12s + & 16t = 64 \end{matrix}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Determine the angle between the planes $2x + y + 7 = 14$ and $x + y + 2z = 7$ (3 marks)

b) Find the equation of the plane through the points (4, -3, 1), (-3, -1, 1) and (4, -2, 8) (4 marks)

c) Solve the homogenous system of linear equation below (4 marks)

$$\begin{aligned} 3r + 5s - 4t &= 0 \\ -3r - 2s + 4t &= 0 \\ 6r + s - 8t &= 0 \end{aligned}$$

d) Find the area of triangle with vertices at $P(2,5,7), Q(4,2,-1), R(3,6,4)$ (4 marks)

e) Determine the value of y if $\begin{vmatrix} y-3 & -2 & 0 \\ 1 & y & -2 \\ -1 & y-1 & y-1 \end{vmatrix} = 0$ (5 marks)

QUESTION THREE (20 MARKS)

a) The 3 x 3 matrix A is given below $A\begin{pmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ 1 & 0 & 3 \end{pmatrix}$

(i) Find the inverse of A (5 marks)

(ii) Hence or otherwise, solve the system of the equation (2 marks)

$$\begin{matrix} 3u & +2v & +w = 7 \\ u & -2v & -w = 1 \\ u & +0v & +3w = 11 \end{matrix}$$

b) Solve the system of equation by Gauss Elimination Method (5 marks)

$$\begin{matrix} a & +3b & +2c = 14 \\ 2a & +b & +c = 7 \\ 3a & +2b & -c = 7 \end{matrix}$$

c) Solve the system of equation by Crammers Rule (5 marks)

$$\begin{matrix} 2x & +5y & +3z = 2 \\ x & +2y & +2z = 4 \\ x & +y & +4z = 11 \end{matrix}$$

QUESTION FOUR (20 MARKS)

a) Let $A = \begin{pmatrix} x & 3 & 6 \\ 1 & x & 1 \\ 0 & 4 & 1 \end{pmatrix}$. Find A^{-1}, in terms of x (6 marks)

b) Find the Rank and Nullity of the system of equation (4 marks)

$$\begin{aligned} a &+2b &-4c+3d- &\ e= &0\\ a &+2b &-2c+2d+ &\ e= &0\\ 2a &+4b &-2c+3d+ &\ 4e= &0 \end{aligned}$$

c) Determine whether the following set of vectors are linearly dependent in R^4 or not $u=(1,2,2,1), v=(2,3,4,1)$ and $w=(3,8,7,5)$ (5 marks)

d) Express $v=x^2+4x-3$ as a linear combination of $e=x^2-2x+5, f=2x^2-3x$ and $g=x+3$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Find all solution of the system $\begin{matrix} 2u+ & 3v+ & 6w=9 \\ 6u+ & 9v+ & 18w=27 \end{matrix}$ (4 marks)

b) Find the area of a triangle with vertices at $P(1,4,6), Q(-2,5,-1), R(1,-1,1)$ (4 marks)

c) Find the equation of a plane passing through (4, 3, -5), (1, -1, 1) and (7, 2, -6) (4 marks)

d) Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear mapping defined by $f(x,y,z) = (2x-2y, 2x+2y+2z)$. Find the kernel of f. (5 marks)

e) Given $A=\begin{pmatrix} 2 & 4 \\ 8 & -6 \end{pmatrix}$. Find a non – zero column vector $u=\begin{pmatrix} x \\ y \end{pmatrix}$ such that $Au=6u$

(3 marks)