# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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**University Examinations 2023/2024**

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE MATHEMATICS AND BACHELOR OF SCIENCE IN STATISTICS

**SMA 3200: CALCULUS III**

**DATE: DECEMBER 2023** **TIME: 2 HOURS**

**INSTRUCTIONS:** *Answer question* ***one*** *and any other* ***two*** *questions*

**QUESTION ONE (30 MARKS)**

a) Replace the following polar equation by equivalent Cartesian equation $r^2 + 2r^2 \cos\theta \sin\theta = 1$ (3 marks)

b) Evaluate $\int_0^\infty xe^{-2x}dx$ and show that the integral converges (3 marks)

c) Verify Rolle's theorem for the function $fx = x + 1/x$ in the interval $\frac{1}{2} \leq x \leq 2$ and find a point c in $\left(\frac{1}{2}, 2\right)$ such that $f'(c) = 0$ (4 marks)

d) Given that $f(x, y)z(70 - 5x + 4y)x + (80 + 6x - 7y)y$ find $f_x$ and $f_y$ and critical point of $f(x, y)$ (6 marks)

e) If $f(x, y) = 2x\sin y + 3y^2 \cos x$, *find* $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$ (4 marks)

f) Evaluate $\int_{x=0}^{4}\int_{y=0}^{x}(x^2+y^2)dydx$ (4 marks)

g) Use the Maclaurin's series expansion to show that

$x\cos x = 1 - x + \frac{x^2}{3} - \frac{x^4}{45} - \frac{2x^6}{945} + ......$ (6 marks)

**QUESTION TWO (20 MARKS)**

a) Find the maximum and the minimum values of the function

$f(x,y) = 2(x^2 - y^2) - x^4 + y^4$ (5 marks)

b) Evaluate $\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} xy^2 dydx$ (5 marks)

c) Evaluate the double integral $\int\int_R \left(6x^2 + 3y^2 + 2\right)dxdy$ where $R$ is the region bounded by the lines $0 \le x \le 1, \quad 0 \le y \le 2$ (6 marks)

d) Evaluate $\int_{1}^{\infty}\frac{1}{x}dx$ (4 marks)

**QUESTION THREE (20 MARKS)**

a) Given that $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ show that $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (5 marks)

b) i. Differentiate between a sequence and a series (3 marks)

ii. By finding the sum of the series (3 marks)

$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ show that the series converges (6 marks)

c) Using the ratio test, show that the series $\sum_{n=1}^{\infty}\frac{1}{n!}$ converges (6 marks)

**QUESTION FOUR (20 MARKS)**

a) State without proof the Rolle's and the mean value theorems (4 marks)

b) Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in the interval $[2,3]$

(5 marks)

c) i. Determine the Taylor series for a function $f$ that has derivatives of all orders throughout interval $I$ at $x = a$ (3 marks)

ii. Find the Taylor series and Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$

(8 marks)

**QUESTION FIVE (20 MARKS)**

a) Define moment man and center of mass of a thin rod along the $x - axis$ (3 marks)

b) A metal rod with one end at the origin and the other end at $x = 10$, thickness from left to right so that its density, instead of being a constant mass per unit length is

$\delta(x) = 1 + \frac{x}{10} kg/m$. Find

i. The moment of the rod about the origin (3 marks)

ii. The mass of the rod (3 marks)

iii. The center of mass of the rod (3 marks)

c) Find $f_x(x,y), f_{xx}(x,y), f_{yy}(x,y)$ *and* $f_{xy}(x,y)$ for the function

$f(x,y) = 4y^3 + x^2 - 12y^2 - 36y + 2$ (4 marks)

d) Locate and classify all the relative extreme of $f(x,y) = xy - x^2 - y^2 - 2x + 4$

(4 marks)