



# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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## University Examinations 2022/2023

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR  
THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR  
THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICAL SCIENCE

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR  
THE DEGREE OF BACHELOR OF SCIENCE IN SCIENCE

SECOND YEAR, SECOND SEMESTER SPECIAL/SUPPLEMENTARY EXAMINATION FOR  
THE DEGREE OF BACHELOR OF EDUCATION TECHNOLOGY

### SMA 3200: CALCULUS III

DATE: AUGUST 2023

TIME: 2 HOURS

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**INSTRUCTIONS:** Answer question one and any other two questions

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#### QUESTION ONE (30 MARKS)

- a) Convert the Cartesian equation  $x^2 + (y - 3)^2 = 9$  into polar equation (4 marks)
- b) Evaluate  $\lim_{x \rightarrow 1} \frac{x^2 - x}{2x^2 + 5x - 7}$  (4 marks)
- c) Determine if the following series converges using the ratio test (5 marks)
- $$\sum_{n=0}^{\infty} \frac{n!}{5^n}$$
- d) Evaluate the integral  $\int \cos^5 x \, dx$  (4 marks)
- e) Determine where the function below is not continuous  $f(x) = \frac{4x+10}{x^2-2x-15}$  (3 marks)
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- f) Find the area of region bounded by the curves  $y = \sin x$ ,  $y = \cos x$ ,  $x = 0$  and  $x = \pi/4$ .  
(5 marks)
- g) The region R, bounded by the curves  $y = x$ ,  $y=x^2$  is rotated about the x-axis. Find the volume of the resulting solid  
(5 marks)

### QUESTION TWO (20 MARKS)

- a) Determine the radius of convergence and interval of convergence of the following power series  
(6 marks)

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n} (x + 3)^n$$

- b) Construct Taylor series expansion for
- (i)  $y = \sin x$  about  $x = 0$  (4 marks)
- (ii)  $y = \cos x$  about  $x=0$  (4 marks)
- c) Find the value  $c$  satisfying the conclusion of the mean value theorem for  $f(x) = x^3 - x^2 - x + 1$  on the interval  $[0, 2]$  (6 marks)

### QUESTION THREE (20 MARKS)

- a) (i) Define the Rolle's theorem (2 marks)
- (ii) Find a number  $c$  satisfying the conclusion of the Rolle's theorem for  $f(x) = x^3 - 3x^2 + 2x + 2$  on the interval  $[0,1]$  (4 marks)
- b) Use integral test to show that  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  is convergent series (4 marks)
- c) Evaluate  $\lim_{x \rightarrow 0} \frac{x^3}{\sin x - x}$  (4 marks)
- d) Use reduction formula to evaluate the integral  $\int \sin^5 x \cos^3 x dx$  (6 marks)



#### QUESTION FOUR (20 MARKS)

a) Evaluate

$$\iint_R (x^2y - 2xy) dA$$

If  $R: 0 \leq x \leq 3, -2 \leq y \leq 0$  (3 marks)

b) Given that  $u(x, y) = 2x^3y^2$ . Show that  $u_{xy} = u_{yx}$  (3 marks)

c) Investigate whether the following integral converges or diverges (4 marks)

$$\int_2^{\infty} \frac{1}{x-1} dx$$

d) Expand  $\sin X$  in Taylor series at  $x = 0$  (4 marks)

e) Find all first and second order partial derivatives of  $f(x, y) = x^4 + 65y - 10xy$  (6 marks)

#### QUESTION FIVE (20 MARKS)

a) Find Cartesian equivalent of the polar equations and identify the graphs of

(i)  $r^2 = 4r \cos \theta$  (4 marks)

(ii)  $r = \frac{4}{2 \cos \theta - \sin \theta}$  (3 marks)

b) Find the mass and centre of mass of triangular lamina with vertices  $(0,0)$ ,  $(1,0)$  and  $(0,2)$  if the density function is  $P(x, y) = 1 + 3x + y$  (8 marks)

c) Evaluate the integral  $\int x^2 e^{3x} dx$  (5 marks)

