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UNIVERSITY EXAMINATIONS 2021/2022

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 3353: RING THEORY

DATE: MAY 2022 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define.
   i. A ring (4 Marks)
   ii. A field (2 Marks)

b) Let $f: \mathfrak{C} \to M_n(\mathbb{R})$, defined by $f(a+bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$. Show that f is a ring homomorphism. (5 Marks)

c) The unit of a ring is unique. Prove. (4 Marks)

d) Find all the units in $\mathbb{Z}_6$. (5 Marks)

e) Divide $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ by $g(x) = x^2 - 2x + 3$ in $\mathbb{Z}_5[x]$ to $q(x)$ and $r\,(x)$ (4 Marks)

f) Find all the prime ideals in $\mathbb{Z}_{12}$. (4 Marks)

g) Consider the ring $\mathbb{Z}_6$. Find the principal ideal of $\mathbb{Z}_6$ generated by 3. (2 Marks)

QUESTION TWO (20 MARKS)

a) Consider the set $S = \mathbb{R}x\mathbb{R}$ of ordered pairs of real numbers. Define the operations + and • on S by: $(a,b) + (c,d) = (a+c, b+d)\ and\ (a,b) \bullet (c,d) = (ad+bc, ac+bd)$
   i. Show that $(S, +, \bullet)$ is commutative with unity. (7 Marks)
   ii. Find the multiplicative inverse of (a, b) and state when it exists. (6 Marks)

iii. Is $(S, +, \bullet)$ a field? Give reasons (s). (2 Marks)

b) Let R be a commutative ring with unity and I be an ideal in R. show that $R/_I$ is an integral domain if I is a prime ideal. (5 Marks)

QUESTION THREE (20 MARKS)

a) Prove that every field is an integral domain. (4 Marks)

b) List all the distinct cossets of $2\mathbb{Z}/8\mathbb{Z}$ and construct the addition and multiplication table for this ring. (11 Marks)

c) Show that $Q(\sqrt{5}) = \{x + y\sqrt{5} \mid x, y \epsilon Q\}$ is a subring of $\mathbb{R}$. (5 Marks)

QUESTION FOUR (20 MARKS)

a) Every finite integral domain is a field. Prove. (6 Marks)

b) Is $x^2 + 3x + 2$ viewed in $\mathbb{Z}_5[x]$ reducible or irreducible? (6 Marks)

c) Find all the zero divisions in $\mathbb{Z}_8$. (4 Marks)

d) If I is an ideal of a ring R, then the canonical map $f: R \rightarrow R/I$ given by $f(x) = x + 1, x \in R$, is a ring homomorphism. Prove (4 Marks)

QUESTION FIVE (20 MARKS)

a) Define an integral domain. (2 Marks)

b) Find all the nilpotent elements in $\mathbb{Z}_9$. (4 Marks)

c) Factorize $x^4 + 3x^3 + 2x + 4$ in $\mathbb{Z}_5[x]$. (8 Marks)

d) Let R be a ring, $I \neq R$ be an ideal of R. prove that.

i. If R is a commutative, then $R/_I$ is commutative. (3 Marks)

ii. If R is a ring with unity 1, then $R/_I$ is a ring with unity 1+I. (3 Marks)