MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.
Tel: 020-2069349, 061-2309217. 064-30320 Cell phone: +254 712524293, +254 789151411
Fax: 064-30321
Website: www.must.ac.ke Email: info@must.ac.ke

University Examinations 2014/2015

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

STA 2401: TIME SERIES ANALYSIS

DATE: AUGUST 2015 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **three** questions*

QUESTION ONE (30 MARKS)

a) Briefly explain the following components of time series:

(i) Secular trend (2 Marks)

(ii) Cyclical variation (2 Marks)

(iii) Seasonal variation (2 Marks)

(iv) Irregular/random variation (2 Marks)

b) Outline four importance of time series (4 Marks)

c) (i) Assuming a four year cycle, calculate the trend of the following data using moving average method

Year	Production
1961	515
1962	518
1963	467
1964	502
1965	546
1966	557
1967	571

1968	586
1969	464
1970	612

(ii) Plot on the same graph the graph of the raw data and the trends and comment about the trend (5 Marks)

d) Define the following models:

(i) iid noise (2 Marks)

(ii) white noise (2 Marks)

e) Show that random walk process is not stationary (8 Marks)

f) $\{x_t\}$ is a zero mean stationary time series with autocovariance function $\gamma(\cdot)$ satisfying $\sum_{h=-\infty}^{\infty} |r(h)| < \infty$. Define the spectral density of $\{x_t\}$ (2 Marks)

QUESTION TWO (20 MARKS)

a) State the conditions necessary for the process to be:

(i) Strongly stationary (2 Marks)

(ii) Weakly stationary (2 Marks)

b) (i) What does it mean for a time series to be causal (2 Marks)

(ii) Define precisely what it mean for a time series y_t to be an AR(1) series (2 Marks)

(iii) What condition will ensure that an AR(1) series will be stationary and causal (2 Marks)

c) Show that the autocovariance function of an AR(1) series is given by

$r_{(4)} = \frac{\sigma^2 \phi^u}{1-\phi^2}$ for $u = 0, \pm 1, \pm 2, ...$ Where ϕ is the parameter of the AR(1) series (5 Marks)

d) Suppose that y_t is a stationary, casual AR(1) series. Describe, as exactly as you can the type of ARMA series which is produced by taking simple different of y_t (5 Marks)

QUESTION THREE (20 MARKS)

a) Define the autocorrelation function (ACF) for a stationary time series (2 Marks)

b) Derive the ACF for the MA(2) series generated by the series scheme

$y_t = \varepsilon_t + \vartheta_1 \varepsilon_{t-1} + \vartheta_2 \varepsilon_{t-2}$ Where ε_t is a white noise series (5 Marks)

c) Derive the ACF for the AR(1) series generated by the scheme

$y_t = \phi y_{t-1} + \varepsilon_t$ Where $|\phi|<1$ and ε_t is a white noise series (5 Marks)

d) For the ARMA (1,2) model $y_t = 0.8y_{t-1} + 0.7\varepsilon_{t-1} + 0.6\varepsilon_{t-2} + \varepsilon_t$

Show that:

(i) $\rho(k) = 0.8\rho(k \sim 1) \quad for\ k \geq 3$ (3 Marks)

(ii) $\rho(2) = 0.8\rho(1) + \dfrac{0.6\sigma_e^2}{\gamma(0)}$ (3 Marks)

e) Describe in general terms, the behaviour you would expect to see the estimated ACF computed from the set of observations $y_1, \ldots\ldots\ldots\ldots y_T$ generated by the random walk model

$y_t = y_{t-1} + \varepsilon_t$ (2 Marks)

QUESTION FOUR (20 MARKS)

a) Explain what it means for a time series y_t to be stationary and mixing (2 Marks)

b) Define the power spectrum for a stationary, mixing time series and describe what the power spectrum reveals abort the time series. (2 Marks)

c) Compute the power spectrum for a white noise ε_t with $E[\varepsilon_t] = 0$ and $\text{var}[\varepsilon_t] = \sigma^2$ (3 Marks)

d) Explain what it means for an operation carried out on a time series to be a linear time invariant filter (3 Marks)

e) Show how to write the transfer function of a linear filter as a function of the filter coefficients (1 Mark)

f) The seasonal differencing filter transforms $y_t - y_{t-s}$ (Simple differencing has s=1). compute the transfer function of this filter (2 Marks)

g) The seasonal summation filter transform the series $y_t\ to\ y_{t-1} + y_{t-2} + \ldots\ldots + y_{t-s+1}$. Show that the second difference filter is equivalent to first applying the seasonal summation operation and the simple differencing (3 Marks)

h) Using the results obtained in (g) or otherwise, compute the transfer function of the seasonal summation (4 Marks)

QUESTION FIVE (20 MARKS)

The following data set gives the number of accidental deaths occurring monthly in the United States during 1973-1978

a) In order to investigate possible non-stationarity of the series, the following three plots were produced form the series. What kind of effects do the plots show and how would these effects influence the analysis of the series (4 Marks)

b) The time series is rear into an R data set called deaths and the following statements are issued to R

```
>acf(diff(diff(deaths, 12),1), 24)
>pacf(diff(diff(deaths, 12)1), 24)
```

Describe in details exactly what these statements do (5 Marks)

c) The following graphs are produced as a result of running the R statements above. Explain what kind of model structure you think that the graphs indicate is appropriate for modeling this series (4 Marks)

d) Using operator notation, write down the complete model which should be filled to the accident data series (2 Marks)

e) A time series model is fit to the deaths series and the following result and plots obtained. Does the model fit well? Give reasons (5 Marks)

Coefficients

	PARAM 1	PARAM 2
	-0.4264	-0.5584
s.e	0.1226	0.1787

sigma ^2 estimated 99480

log likelihood =-425.53

AIC=857.06