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University Examinations 2014/2015

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF ACTUARIAL SCIENCE

STA 2290: FINANCIAL MATHEMATICS II

DATE: APRIL 2015

TIME: 2 HOURS

INSTRUCTIONS: Answer question *one* and any other *two* questions

QUESTION ONE (30 MARKS)

- a) Explain why the term maturity of a bond is important with respect to bond valuation (3 Marks)
- b) An economist's model of interest rate indicates that the year spot rate of interest is $0.1(1 + e^{-0.1t})^{-1}$. According to this model, what is the price of a 10 year zero coupon bond redeemable at par (4 Marks)
- c) A fixed interest security with a 6% annual coupon payable half-yearly in arrears is purchased at a price that gives a gross effective yield of 10% pa by an investor who is subject to capital gains tax at 30%. It is redeemable at par after 15 years. Calculate the amount of capital gains tax payable per 100/= nominal (3 Marks)
- d) Define an interest rate swap and describe briefly the two kinds of risk facing each counterparty in the swap (3 Marks)
- e) Explain the main differences between futures and forward contracts (5 Marks)
- f) The force of interest, $\delta(t)$ is a function of time t , measured in years and is given by

$$\delta(t) = 0.03 - 0.005t + 0.001t^2 \quad 0 \leq t \leq 10$$

- i) Calculate the equivalent constant force of interest per annum for the period $t=0$
(4 Marks)
- ii) Calculate the accumulated value at time $t=7$ of an investment of £250 at time $t=0$
plus further investment of £150 at time $t=5$ (4 Marks)
- g) An investor purchases a 5 month forward contract on 1 January 2004 to buy 1000 shares at the end of the contract. The price of a share on 1 January 2004 is shs.50. Dividends are received continuously and the dividend yield is 6% pa. The risk free rate of interest is 4% pa effective and no arbitrage. Calculate the forward price (4 Marks)

QUESTION TWO (20 MARKS)

- a) An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains. Calculate the purchase price on the bond per £100 nominal to provide the investor with a rate of return of 8% per annum effective (8 Marks)
- b) A 91-day government bill is discounted at a simple rate of discount of 10% pa. Calculate the price paid for £10000 nominal that will be redeemed at par (3 Marks)
- c) An investor who is liable to income tax at 20% but is not liable to capital gains tax wished to earn an effective rate of return of 5% per annum. A bond bearing coupons payable half-yearly in arrears at a rate of 6.25% per annum is available. The bond will be redeemed at par on a coupon date between 10 to 15 years after date of issue, inclusive. The date of redemption is at the option of the borrower. Calculate the maximum price that the investor is willing to pay for the bond (5 Marks)
- d) An investor purchased a ten-year bond paying coupons of Kshs.36 per annum for Kshs.800. Suppose the bond will be redeemed at Kshs.940. Calculate the net yield achieved by the investor (4 Marks)

QUESTION THREE (20 MARKS)

- a) The 3, 5 and 7-year spot rates are 6%, 5.7% and 5% pa respectively. The 3-year forward rate from time 4 is 5.2% pa. Calculate f_3 (3 Marks)

- b) The force of interest $\delta(t)$ is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 + 0.005t & 0 \leq t < 6 \\ 0.16 - 0.015t & 6 \leq t < 8 \\ 0.04 & 8 \leq t \end{cases}$$

- i) Derive expressions in terms of t for the accumulated amount at time t of an investment of I at time 0
- ii) Calculate the value at time 0 of £100 due at time 9 (13 Marks)
- c) Explain four reasons as to why interest rates vary with the term of an investment (4 Marks)

QUESTION FOUR (20 MARKS)

- a) Differentiate between preference and ordinary shares in terms of security and marketability of the two assets (2 Marks)
- b) A pension fund expects to make payments of shs.100,000 per annum at the end of each of the next five year. It wishes to immunize these liabilities by investing in two zero coupon bonds which mature in five years and in one year respectively. The rate of interest is 5% per annum effective
- i) Show that the present value of the liabilities is shs.432,948 and that the duration of the liabilities is 29 years (6 Marks)
- ii) Calculate the nominal amounts of the two zero coupon bonds which must be purchased if the pension fund is to equate the present value and duration of assets and liabilities (6 Marks)
- iii) Calculate the convexity of the assets and liabilities and comment on whether you think Redington's immunization has been achieved (6 Marks)

QUESTION FIVE (20 MARKS)

- a) The effective annual rates of return that will apply during each of the next n years might be i_1, i_2, \dots, i_n where $i_k, k = 1, 2, 3, \dots, n$ are random variables with the following discrete distribution

$$i_k = \begin{cases} 0.06 & \text{with probability } 0.2 \\ 0.08 & \text{with probability } 0.7 \\ 0.10 & \text{with probability } 0.1 \end{cases}$$

- i) Calculate the mean, j , and standard deviation, s , of i_k (5 Marks)
- ii) Calculate the mean and variance of accumulated value at the end of 5 years of an initial investment of £5000 if the returns of the investment are assumed to conform to fixed interest rate model (7 Marks)
- iii) Calculate the accumulated value at the mean rate of interest (2 Marks)
- b) A man now aged exactly 50 has built up a savings fund of £400,000. In order to retire at age 60, he will require a fund of at least £600,000 at that time. Calculate the probability that if he makes no further contributions to the fund, he will be able to retire at age 60. Assume that annual growth rates vary independently from year to year and have a log normal distribution with parameters $\mu = 0.075$ and variance $= 0.1^2$ (6 Marks)