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University Examinations 2014/2015

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

SMA 2407: FUNCTIONAL ANALYSIS

DATE: APRIL 2015 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) (i) Define metric space (4 Marks)

(ii) Give four examples of metric spaces (4 Marks)

b) Show that $d(x, y) = \sqrt{|x - y|}$ define a metric on the set of all real numbers (4 Marks)

c) (i) If we let (X, d) be a metric space, define a Cauchy sequence (2 Marks)

(ii) Prove that any convergent sequence in a metric space is Cauchy sequence (4 Marks)

d) The real line interval (0,1) with the usual metric is not a complete space. Prove (4 Marks)

e) (i) Define nested sequence in a compact space (2 Marks)

(ii) State Cantor's intersection thereon (2 Marks)

f) (i) Define compactness in a metric space (2 Marks)

(ii) Show that (0, 1] is not compact (2 Marks)

QUESTION TWO (20 MARKS)

a) Define a discrete metric space (3 Marks)

b) State the Eudidean metric on $\mathbb{R}^2$ and illustrate the same on the plane (4 Marks)

c) (i) Let X be the set of all ordered triples of zeros and ones. Show that X consists of eight elements and then list them (5 Marks)

(ii) Give a definition of the metric of d of X (2 Marks)

(iii) What is the name $d(x, y)$, the distance between x and y (1 Mark)

d) State the Eudidean metric on $\mathbb{R}^3$ (5 Marks)

QUESTION THREE (20 MARKS)

a) (i) Define sequential compactness in a metric space (3 Marks)

(ii) [0,1] is compact, but (0,1) is not compact show why (3 Marks)

(iii)Prove that any closed interval [a,b] is compact (6 Marks)

b) Suppose that M is discrete metric space and N any metric space, prove that any function $f : M \to N$ is continuous (8 Marks)

QUESTION FOUR (20 MARKS)

a) (i) Define contraction in a metric space X (2 Marks)

(ii) State and prove the Banach fixed point theorem (contraction theorem) (4 Marks)

b) Define Equicontinous function in a metric space (4 Marks)

c) State the Arzela-Ascoli theorem in compact metric space (4 Marks)

QUESTION FIVE (20 MARKS)

a) (i) If we let A be a non empty subset of a metric space (X, d). Define the diameter of A (2 Marks)

(ii) Let (X, d) be a complete metric space. If (f_n) is a sequence of non empty closed subsets of X such that $F_n / I \subseteq F_n$ for all $n \in N$ and $(diam(F_n)$ converges to O, prove that $\cap_{n=1}^{\infty}, F_n$ is a singleton (10 Marks)

b) (i) Define a uniformly continuous function f from a metric space (X, d) into a metric space (Y, p) (2 Marks)

(ii) Prove that a uniformly continuous function maps Cauchy Sequences into Cauchy Sequences (6 Marks)